A COMPARISON OF THE DOUBLE–CIRCULAR–ARC–GEAR DRIVES WITH STANDARD INVOLUTE GEAR DRIVES

by

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ABSTRACT

In recent years the Chinese and others have taken the circular–arc–gear and developed the approach into the double–circular–arc–gear. They have now developed the double–circular–arc–gear for use in several industrial gear drive systems. Some of these units are being marketed as pumping units in the oil well pumping market. There is therefore a need to compare the double–circular–arc–gear drives for oil well pumping units with the standard involute gear pumping units. The American Petroleum Institute standard specification 11E and the Chinese developed standard for double–circular–arc–gears were used to compare the two gear systems.

The double–circular–arc–gear pumping units were shown to have torque capacities that were comparable to or better than the involute gear units based on the data developed by the Chinese for the various strength ratings of the circular–arc–gears.

INTRODUCTION

The circular arc gears have been around since the late 20's when they were invented by E. Wildhaber (1926). However, very little work on an application of circular arc gears has taken place in the United States. The Russians, also developed a patent (Novikov) for circular arc gears. The Boeing Vertol Company investigated circular arc gears for helicopter applications, Lemanski et al (1968) and found that they had the advantage of increased surface contact capacity, less sliding motion and higher efficiency with about the same bending capacity as involute gears. They also found that they are very sensitive to center distance variations. Westland Helicopter Company developed and put into production a helicopter gear box using circular arc gears and have shown good results with the transmission, B. Shotter (1977). The Russians have used circular arc gearing for many gears mainly in industrial applications. In recent years the Japanese, Russians and Chinese have conducted considerable research and development work on improving the circular arc gear by using a double circular arc tooth form or a combined circular arc and involute profile called simarc. Ariga and Nagata (1981). This latest design approach has shown good results in improving the bending strength and reducing the effects of center distance variations on the circular arc gear. The Chinese and Russians have now developed the double circular arc gear to the point where they are producing different drive units for commercial and military applications. Pyshkin and Saigin (1978). There has been considerable research and development work conducted on circular arc and double circular arc gears at the Harbin Research Institute in China. Chen, Chen and Chen (1988) Ref. They have developed the hertzian and bending strength equations for double circular arc gears. In addition, the Chinese have developed several standard tooth forms which are used for the manufacture of these commercial double circular arc gear drive units Chinese Standard (1981).

DESCRIPTION OF GEAR SYSTEMS

With the involute system of gearing, a gear drive can be either spur, helical or double helical. Usually higher speed gear drives are helical or double helical to reduce gear vibrations and noise. With circular arc and double circular arc gears, the drives have to be either helical or double helical since the gears contact at only one point on the profile for circular arc and two points for double circular arc gears. The contact point moves along the helical angle as the gears rotate.

Figure 1 is a cross section of the circular arc gear and shows the different tooth shape between the gear and pinion. It has been shown that the circular arc gear is sensitive to center distance variations because the two arcs do not match when they are not located at their designed center distance. This can be relieved somewhat by making the concave arc slightly larger than the convex arc.

The simarc gear, figure 2, developed in Japan, Ariga and Nagata (1981) is designed to have some of the benefits of the circular arc gear and to remove the sensitivity to the center distance variation. This gear design has been tested by various groups in Japan and shown to have good results.

In Russia and China the double circular arc gears, figure 3, have been developed and are now being used as standard gears in various applications. In the double circular arc gear, there are two points of contact on the profile that move along the helix angle during gear rotation. This two point contact on the profile gives considerable improvement in the load capacity for these gears in both bending and hertzian contact stress.

RATING METHODS FOR DOUBLE CIRCULAR ARC GEAR SYSTEMS

The load capacity equation for these gears was developed at the Harbin Research Center in China, Chen, Chen and Chen, (1988) and . figure 4 is a plot of the tooth form factor Y_f developed at Harbin Research Center for the bending strength for the double circular arc gears.
They have developed experimental and analytical values for the Y_f factor and have chosen to use the more conservative analytical values which were about 8 to 12% less than the experimental values.

The Chinese have developed the following equations for the pitting resistance torque rating and the bending strength torque rating for the pinion of a double circular arc gear unit.

$$\text{Pitting } T_p = \frac{2 \quad \mu_E + K_{H\Delta\epsilon}}{K_A \quad K_V \quad K_1 \quad K_{H2}} \left[\frac{1}{d_p}\right]^3 \left[\frac{Z_1 \quad \sigma_{Hp}}{Z_E \quad Z_u \quad Z_\beta \quad Z_a}\right]^{1/.73}$$

$$\text{Bending } T_b = \frac{2 \quad \mu_E + K_{F\Delta\epsilon}}{K_A \quad K_V \quad K_1} \left[\frac{1}{d_p}\right]^3 \left[\frac{Z_1 \quad \sigma_{Fp}}{Y_E \quad Y_\mu \quad Y_\beta \quad Y_F \quad Y_{end}}\right]^{1/.86}$$

where

$\mu_E =$ Round number for overlap ratio

$K_{H\Delta\epsilon}$, $K_{F\Delta\epsilon}$ = Influence factor for coincidence mantissa

K_A, $K_{V,}$ K_1 = Application, velocity & load distribution factor between each contact trace

K_{Hz} = Load distribution factor on one contact trace

d_p = diametral pitch

Z_1 = Number of teeth pinion

σ_{Hp} = Allowable contact stress

σ_{Fp} = Allowable bending stress

Z_E, Y_E = Elastic coefficient

Z_u, Y_u = Gear ratio factor

Z_β, Y_β = Helix factor

Z_a = Contact arc length factor

Y_F = Tooth profile factor

Y_{end} = Tooth end factor

These equations do not include the gear ratio as used in API specification 11E and must be multiplied by the gear ratio to obtain the equivalent torque values obtained by equations 1 and 14 from the API specification 11E, dated 6–1–88 for involute units. For the static torque rating, equation 23 from API specification 11E for involute gears is also used for the double circular arc gear units.

$$T_{as} = \frac{D}{2}\,\frac{J}{p_d}\,\frac{F}{K_{ms}}\,S_{ay}\,K_y$$

The J factor for equation 23 must be determined for the double circular arc gears using the Y_f factor developed by the Chinese and the method of AGMA 226.01 dated 8–70. When using AGMA 226.01 some simplifying estimates or assumptions must be used since there are two load points on the double circular arc gear compared to one load point on the involute gear used in the AGMA standard. For the ratings presented here it was assumed that the total load was carried on the outer circular arc of the gear. This would provide a conservative or reduced static torque capacity rating compared to an actual case where two load sharing points occur on the double circular arc gears.

RESULTS AND DISCUSSION

There are several double circular arc gear units that were designed and manufactured for oil well pumping systems. There is, therefore, a need to compare the torque capacity of the double circular arc gear drives, fig 5, with a comparable involute gear drive. The American Petroleum Institute specification 11E is the standard used by the petroleum industry for rating the involute gear drives. This standard uses the AGMA rating methods to determine the torque capacity of the gearboxes. Since the

United States industry has not developed a standard for the double circular arc gear systems, it is necessary to use the standards developed by the Russians and Chinese to evaluate the torque rating for bending and pitting of the double circular arc gear units. The static torque rating of the double circular arc units can be determined using the API specification 11E with the Chinese developed Y_f factor and the AGMA graphical method to determine an approximate stress correction factor, K_f and J factor.

Using the above mentioned approach, a comparison was made between a standard involute size 912 oil well pumping unit gear box and a size 912 double circular arc oil well pumping unit gear box. The allowable stresses used in the various equations were determined from the material hardness data supplied by the Chinese. Different values were used for the involute and double circular arc gear units since the double circular units were manufactured with a lower hardness material. Table 1 lists the material properties and various torque ratings for the standard involute and double circular arc gearboxes. The pitting torque capacity for the double circular arc gears was approximately 20% higher than the involute even with the lower hardness material. This would be expected since the double circular arc provides a lower contact stress. The bending strength torque for the double circular arc gear was approximately 8% lower than the involute. The static torque capacity for the double circular arc gear was somewhat higher than the involute gears with a very conservative approach in the determination of the J factor for the double circular gears.

CONCLUSIONS

A 912 size oil well pumping unit with involute type gears was compared with a similar size and type pumping unit with double–circular–arc type gears. The API spec 11E was used to determine the torque rating for the involute unit and the static torque rating for the circular–arc unit. The Chinese developed standard was used to determine the pitting and bending torque rating for the circular–arc unit. The material used in the involute unit had a little better strength than the material used in the circular–arc unit because of its higher hardness. The double–circular–arc–gear unit had a higher pitting and static torque rating than the involute unit. The bending torque rating of the circular–arc unit was a little less than the involute unit but well above the required torque rating for a size 912 unit.

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TABLE 1

DOUBLE REDUCTION DOUBLE HELICAL GEAR BOX

912 SIZE OIL PUMPING UNIT

INVOLUTE		HARDNESS BHN	GEOMETRY FACTOR J	TORQUE RATING LB-IN PITTING	BENDING	STATIC
FIRST REDUCTION	GEAR	270	.53	265191	440591	2000490
	PINION	310	.45		410206	2048740
SECOND REDUCTION	GEAR	270	.53	1063157	1433801	6561713
	PINION	310	.46		1364584	6869314
DOUBLE CIRCULAR ARC						
FIRST REDUCTION	GEAR	245	.77	330936	355000	2289129
	PINION	285	.88		381532	3209678
SECOND REDUCTION	GEAR	245	.89	1236000	1347176	6855270
	PINION	285	1.16		1262540	9678030

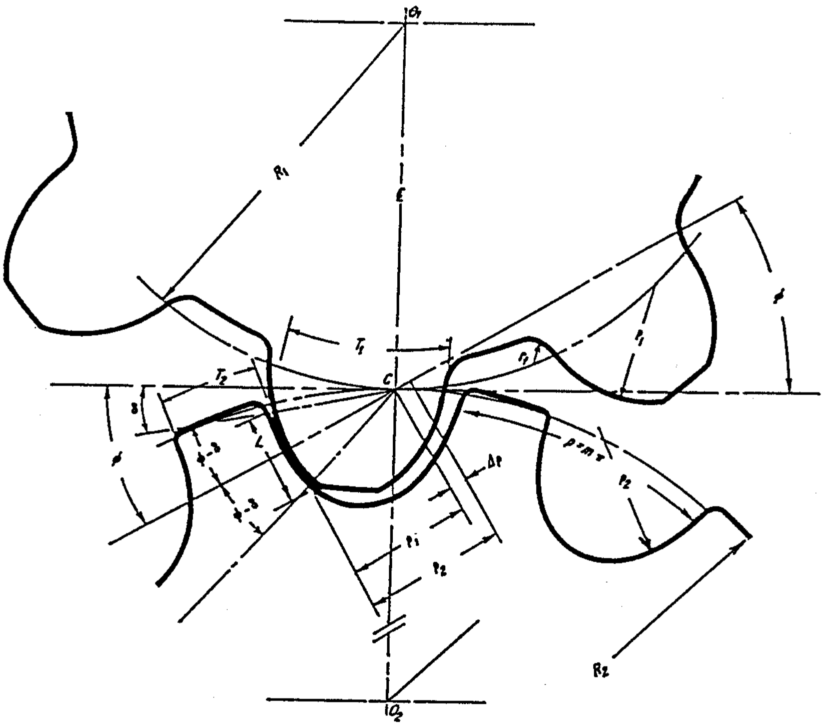

Fig. 1 Wildhaber Novikov (Conformal) Gear

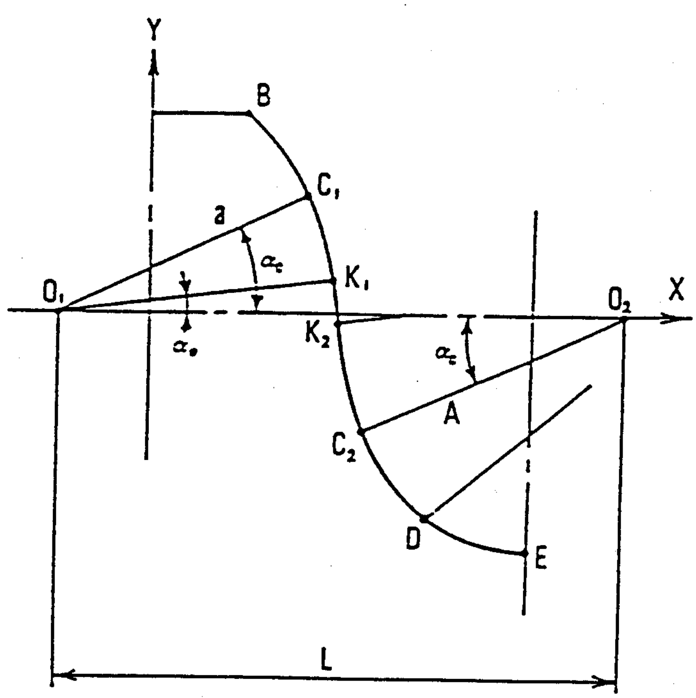

Fig. 2 Tooth Profile of a Simarc Gear
Combined Involute and Circular–Arc

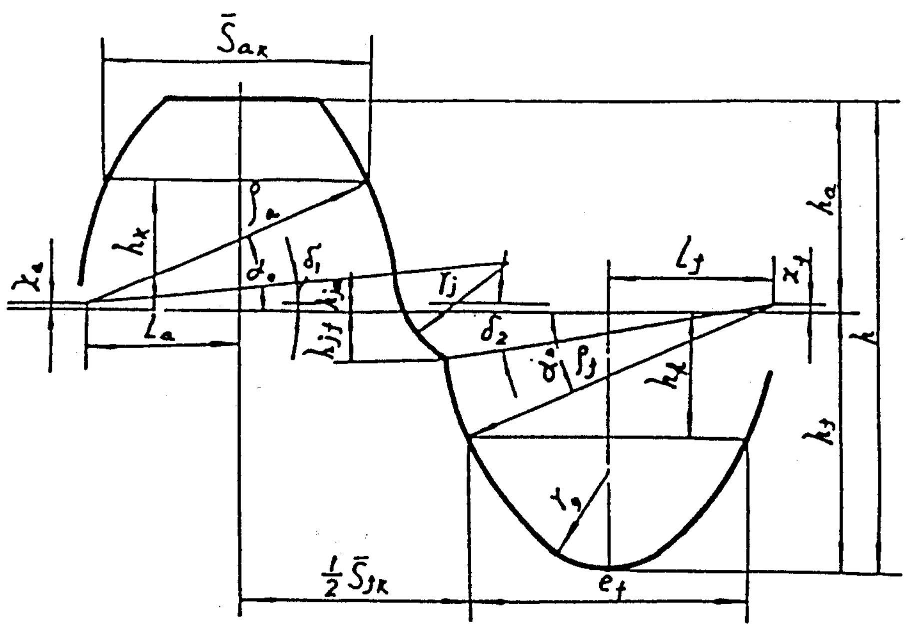

Fig. 3 Basic Rack Tooth Profile of Double-Circular-Arc Gear Cutting Hob

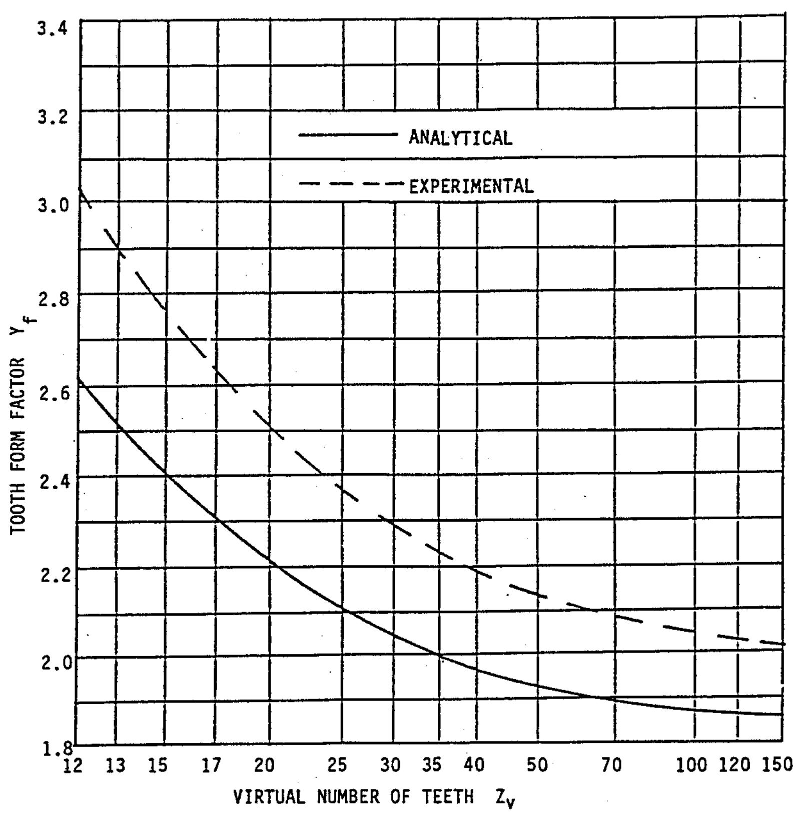

Fig. 4 Tooth Form Factor Y_f for Double-Circular-Arc Gear.

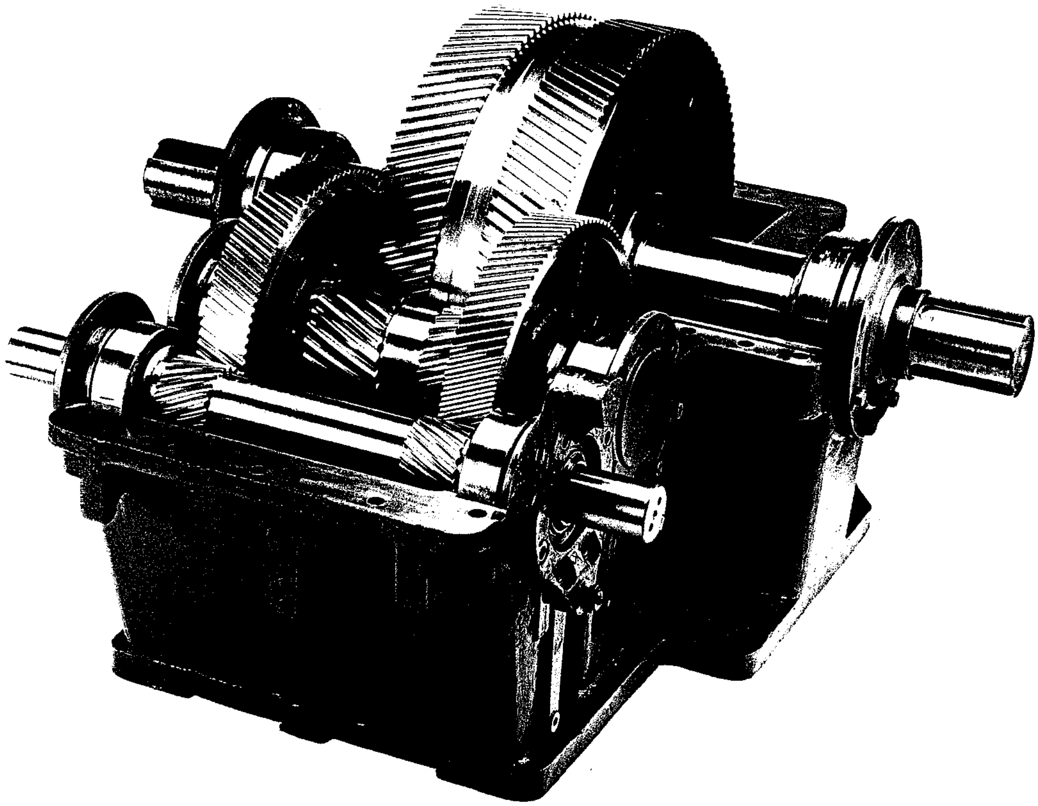

Fig. 5 Double Reduction, Double Helical, Double–Circular–Arc Gear Oil Well Pumping Unit.